Module: Animal Physiology: Size and Surface Area in Animal Physiology

Module Content:
The major goals of the module are for students to a) be exposed to the overwhelming importance of size in an animal's life; b) understand the relationships between surface area, volume, and size; and c) see how the relationship between surface area and its volume is fundamental to the operation of many animal systems. The module includes simple calculations of surface area, an introduction to the mathematical relationship between size and heat loss/metabolic rate, and a series of questions exploring the relationship between surface area and organ function. There is also an opportunity to students to extend this understanding to the cellular level in optional additional exercises.

The module is designed to be implemented in a 50-minute classroom session with a preparatory assignment for students to complete and turn in at the beginning of the session and optional follow-up homework questions. The module is designed for first-year biology majors in an introductory biology course. The role of size and especially surface area to volume ratio are critical to nearly all animal systems as well as at the cellular level, but are usually not dealt with directly in lecture, so this module provides an opportunity for students to connect information from different systems using this theme, as well as develop the module-specific skills and extend the skills they developed in the Introduction to Mathematical Modeling in Biology Module, if that module is used previous to this one.

Students need only to have basic mathematical skills, such as algebra and ability to construct a graph, to complete this module. Specifically, the module includes:
1. A preparatory assignment to prepare the students for the in-class assignment
2. An in class teamwork activity including introductory questions about the role of surface area in the movement of heat, the use of a mathematical model of the relationship between an animal’s size and its metabolic rate, and a few questions about the importance of high surface area:volume ratios in the lungs of animals. A Powerpoint file is included, though questions and other content could be written on a board and/or given verbally, or transferred to a worksheet.
3. Optional accessory problems to use as a homework assignment, dealing with the role of size and surface area at the cellular level.
4. Questions to use for assessment of the concepts and skills in this module.

Competencies Addressed:
E1: Apply quantitative reasoning and appropriate mathematics to describe or explain phenomena in the natural world.

Specific Learning Objectives:
-- Demonstrate quantitative numeracy and facility with the language of mathematics.
-- Interpret data sets and communicate those interpretations using visual and other appropriate tools.
-- Make inferences about natural phenomena using mathematical models.
-- Quantify and interpret changes in dynamical systems.

**E7**: Explain how organisms sense and control their internal environment and how they respond to external change.

**Specific Learning Objective**:
-- Explain maintenance of homeostasis in living organisms by using principles of mass transport, heat transfer, energy balance, and feedback and control systems.

*Is there a physics competency that applies?*

**Preparatory Worksheet for the Size and Surface Area Module**

**To be handed in (or posted electronically) at the beginning of class.**

Name: _____ KEY ______________

**Surface Area to Volume Ratios Worksheet**

**Key Concept Questions:**

1. Fill out the table below by calculating surface area, volume, and surface area to volume ratio (SVR) for each of the cubes.

<table>
<thead>
<tr>
<th>Cube</th>
<th>Surface Area</th>
<th>Volume</th>
<th>SVR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1</td>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2 2</td>
<td>24</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>
2. Describe the relationships between surface area, volume, and SVR (y-axis) to increasing cube size (x-axis) in a graph and in a few sentences.

As cube side length increases, surface area and volume increase in an exponential fashion while the SVR decreases in an exponential fashion.

3. a. Animals can be thought of as simple three-dimensional shapes. Suppose a black-tailed prairie dog is represented as a rectangular prism with units 2x1x1 (length x width x height) and an American bison is represented as a rectangular
prism with units 24x6x14. Fill out the table below by calculating surface area, volume, and SVR for the bison and prairie dog.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Surface Area</th>
<th>Volume</th>
<th>SVR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1128</td>
<td>2016</td>
<td>0.56</td>
</tr>
</tbody>
</table>

b. Mammals maintain a relatively constant body temperature. This is an energy-intensive task especially in the winter, when animals lose heat to the cold air. Because animals lose heat proportional to their surface areas… [etc. do problem here; determine heat loss per unit volume (size, assuming volume is proportional to mass) – total heat loss (joules) is larger in the bison but heat loss/gram is much higher in the prairie dog]

In Class Exercises:
There is no handout; questions and concepts are projected as a slideshow or written on the board by the instructor. The .ppt file is provided as part of the module. Students get in groups of 4-5, and are shown a question in a slide. Each student group writes down their answer on their white board or large paper, and at instructor’s signal, they all hold up their boards/papers. Based on the answers on the papers, the TA moves forward or leads a discussion. The first two questions are essentially warm-up questions to help students become comfortable with the material.

1. Why are mittens warmer than gloves? **Lower SVR means heat loss is slower**

2. Why do muffins cook faster than bread, made from the same batter? [show muffin tin and bread pan on slide/board] **Higher SVR means heat gain is faster**

3. As you saw in the homework, small animals lose heat faster, *for their size*, than large animals. Partly because of this, small animals have to eat more *for*
their size than large animals. This shows how much a vole and a rhino have to eat in a week compared to their body size:

How much animals have to eat depends on how fast they are using up energy— their **metabolic rate**. Here is the equation for figuring out an animal’s total metabolic rate (not the metabolic rate per gram):

\[ M = W^b \]

where \( M \) = total metabolic rate, \( W \) = mass of the animal in grams, and \( b = 0.75 \) for all groups of animals.

A mouse weighs 20g and a small elephant weighs 2,000,000g.

a) If \( b \) were 1, then \( M = W \). Plot the mouse and the elephant on a graph with size on the x axis and total metabolic rate on the y axis. Based on this value for \( b \), an elephant’s metabolic rate would be \( _{100,000}x _______ \) times higher than a mouse’s.

b) Actually, from looking at data from real animals, scientists have noticed that the points don’t fall on that “\( b = 1 \)” line. Instead they seem to follow \( M = W^b \) where \( b \) isn’t 1, but instead is 0.75. Using this new value for \( b \), calculate the metabolic rate for the real mouse and elephant and plot them on your graph. Based on your calculations:

an elephant’s metabolic rate is \( _{5000}x _______ \) times higher than a mouse’s.

Instructors, point out that this means that the elephant has a higher TOTAL metabolic rate—5000x higher!—it must eat 5000x more!—but that the mouse has a higher per-gram/ weight-specific metabolic rate (what people think of as a metabolic rate) than the elephant.
c) Plot these three animals on a graph: a bat (10g), a raccoon [?20,000g?], and a camel [?1,000,000 g?]. Considering that the higher an animal’s total metabolic rate is, the more it has to eat, which of these animals probably couldn’t exist if metabolic rate scaled linearly with size (if b were 1)?

The camel couldn’t exist if b were 1, because no animal can eat that much food.

4. Mammals have extraordinarily high metabolic rates, so they need a lot of surface area to provide oxygen and void carbon dioxide (reactants and products of aerobic respiration). To provide this surface area, lungs branch many times, like a tree, and end in little sacs called alveoli, where oxygen and carbon dioxide are exchanged between the air and blood. If the lungs did not branch at all, there would be one alveolus. If they branched once, there would be two alveoli.

a) How many alveoli would there be if the lungs branches twice? $2^2 = 4$

b) What if the lungs branched 8 times? $2^8 = 256$

c) What if—as is actually the case—the lungs branched 29 times? [Please don’t expand your answer—leave it as an exponent.] $2^{29} (536,870,000)$

5. Each alveolus is 0.07mm$^2$ in area. What is the total alveolar (surface) area and the surface (alveolar) area to volume ratio (assume a lung volume of 2L, or 2,000,000 mm$^3$) of the lung of a person? How does this SAVR compare to that of the blocks and animals you calculated in the homework?

Area = 37,580,963 mm$^2$
SAVR = 37,580,963/2,000,000 = 18.8, higher than the 1x1 cube or the prairie dog.

6. Instead of having lungs that transfer oxygen into the bloodstream, insects have tubes carrying air directly to each part of their bodies:

Given this system, why are there no flies as big as mice?

Diffusion rate is very slow over long distances (is proportional to the inverse square of distance*), so it would take roughly forever for oxygen to diffuse to the inside of an animal as big as a mouse. [They might not know, but use this to explain why large animals have respiratory and circulatory systems.]

*Another thing that can be modeled by students if time allows.

Optional Accessory Problem Sets for Homework:

(Alternatively, these could be done in class, if time allows.)

1. Essential molecules pass into the cell via the cell membrane. What parameter of a cell determines how much (total) a cell can absorb: surface area, volume, or mass? Surface area
2. The cell must obtain essential molecules so that the organelles of the cell can function. What parameter determines how many organelles the cell is providing for: surface area, volume, or mass? **Volume**

3. Assume that a cell requires 2.5 units of surface area for every 1 unit of volume to provide enough essential molecules for survival. Which cell sizes shown above [**repeat figure from Q1 of homework assignment**] are physically possible? Cell sizes 1 and 2 are physically possible.

4. Why are single cells limited to such a small size?
   *Because a cell must be able to absorb enough nutrients to provide for its content; in other words, it must have a sufficiently high SAVR. As cell size increases, volume increases disproportionately to surface area and the cell is unable to absorb enough nutrients to provide for its content.*

5. [Or could be an assessment question] Why do the cells lining the small intestine each have numerous microvilli projecting from their cell membranes?

**Instructions for Implementation by TAs:**
Collect homework. Have students break up into groups, ideally of 4-5 students each, and give each student group a simple dry-erase board (available for about $3 apiece at office-supply stores), mini-chalkboard, or large piece of paper, and a marker. Ask each question in the module, one at a time, by projecting the slide with the question or writing it on the board. Some questions also have introductory content, which is provided on the slides. Give the students a few minutes with each question or sub-question—**not a long time**, no more than 3-4 minutes per question and some questions need only a minute, such as the first two questions. The shorter the interval the higher the level of energy and interest in the room. As the students work, circulate and assist them (without giving them the answer, of course). At the end of the time period for the question, announce that there are 10 seconds remaining, then ring a bell or use some other pre-agreed signal,* and at the signal, all student groups hold up their white boards with their answers. Use the boards as a basis for
discussion if answers differ. If most student groups have the right answer, move on quickly to the next question. The ppt file is provided as part of the module.

Alternatively, the questions can all be given together to each student group as a worksheet. This sounds like less work and stress for the TAs/instructors, but the one-at-a-time-method keeps everyone on track, energized and having fun. Try it!

Assessment Guidelines:

1. Why do you unload the forks first—before the knives and spoons-- from the silverware rack of a just-opened hot dishwasher?  
Because they have a higher SAVR and so cool down faster, allowing you to touch them without harm.

2. Amphibians are able to breathe across their skin as well as in their lungs. During the mating season, male Hairy Frogs grow filamentous projections on their legs and sides. What could be the purpose of these projections? Why would a male need these projections during the breeding season and not throughout the year (keep in mind the throat pouches of frogs are some of the most energy demanding tissues of the animal world)?

   The added projections greatly increase surface area to volume ratio. Male frogs exert a huge amount of energy in the mating season calling for mates and mating. Increased gas exchange allows for a higher metabolic rate and higher overall activity level that likely results in a higher relative fitness in comparison with individuals with smaller projections or fewer projections.

3. You have two dogs, one that weighs 30 pounds and one that weighs 60 pounds. How much does the big dog eat compared to the small dog?  
a) it eats the same amount  
b) it eats less  
c) it eats more, but not twice as much  
d) it eats twice as much  
e) it eats more than twice as much

This gets at a common misconception that because small animals have high per-gram metabolic rates, they actually eat MORE than large animals. The meadow vole-white rhino picture can actually contribute to this misconception.
4. In discussion section you calculated the surface area to volume ratio (SAVR) of a human lung. Humans are endotherms and reptiles are ectotherms. Based on this, predict the SAVR of a reptile’s lung. Why exactly would they differ?

Endothermy requires tons of energy, so endotherms have a much higher metabolic rate/need for oxygen than ectotherms. In order to obtain the necessary oxygen, they require a high SAVR. For a reptile, this high SAVR would be a waste of energy to create and maintain; they should, and do, have far lower SAVRs than mammals.