Plastic Water Bottle Design –
What's the Best Shape?

Grade Level:	**Total Time Required:**
6	3-5periods (50 minute each), approximate

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Lesson Objectives:

Students will be able to:

1. Measure the volume and surface area of objects.
2. Determine surface area to volume ratios.
3. Optimize the surface area to volume ratio (to reduce the amount of plastic used to make the bottles.

Indiana Standards:

6-8.E.1 Identify the criteria and constraints of a design to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.

Next Generation Science Standards:

S-ETS1-1 Define the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.

MS-ET1-4 Develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.

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Mathematics Connections:

6. RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6. EE. 2 Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.

6. EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l wh and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

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Concepts and Vocabulary

Term	***Defined by a scientist or engineer***	***Defined by a student***
Surface area	Total area of faces and curves of a solid object	How much material it takes to make
Volume	Quantity of an enclosed 3-D space	How much it can hold; size
Radius	Half the diameter	Center to outside of circle
Ratio	Relationship between two numbers	One number divided by another
Optimization	Finding the minimum or maximum of a mathematical relation	Finding the best value for a quantity
Surface area	Total area of faces and curves of a solid object	How much material it takes to make
Volume	Quantity of an enclosed 3-D space	How much it can hold; size
Radius	Half the diameter	Center to outside of circle
Ratio	Relationship between two numbers	One number divided by another
Optimization	Finding the minimum or maximum of a mathematical relation	Finding the best value for a quantity

Equipment, Materials, and Tools

Materials		
Plastic bottles (1 Liter, or various sizes)	Paper, card stock	String
Tape	Water (500 ml)	

Tools		
Scissors	Metric rulers (cm scale)	Measuring cups
Funnels	Calculators	Micrometer (optional, to measure thickness of plastic)

Science Content - Basics

Americans use an estimated 29 billion plastic water bottles* per year. Manufacturing these bottles uses about 17 million barrels of crude oil (equivalent to almost one day's use of crude for the U.S.), as well as substantial water and energy consumed in the manufacturing process. It is

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estimated that only about one out of six plastic water bottles is recycled, instead they become part of landfills and oceans and they do not degrade easily.

* A typical school classroom, filled floor to ceiling, could hold about 300,000 one-liter plastic bottles. So, it would take about 100,000 classrooms to hold all of the plastic bottles used in the United States in just one year.

Figure 1. Pile of plastic bottle waste
(*http://kids.nationalgeographic.com/kids/stories/spacescience/water-bottle-pollution/*)

Several manufacturers are trying to reduce the plastic used in water bottles. Thus far, it appears that none of these manufacturers is addressing the optimum shape for bottles of standard volumes (500 ml, 1000 ml, etc.). The design task is to use the ratio of surface area to volume to optimize the shape and size (radius, height, side length, etc.) of bottles to minimize plastic used. Bottle specifics such as ridges, divots, cone opening and caps should be neglected. The students will gain significant practice with calculations (surface area and volume of standard shapes) and a relatively simple optimization problem. The students will also be challenged to consider other constraints and requirements of plastic bottles for consumer use in determining the best design. A follow-up question will be to brainstorm other solutions or strategies to address the issue of plastic bottles, the efficient use of raw materials, and the impact on the environment.

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Lesson Plan #1
Guiding Question – Exploring Volume and Surface Area

Time: One 50 minute class period

Objective: Students will watch a video about the hazards of too much plastic and measure volume and surface area using formulas.

Materials:

- Video link
- 1 liter (or other sizes) plastic water bottle for each group
- Measuring cup (capable of 250, 500, 750, and 1000 ml measures) for each group
- Centimeter rulers
- Calculators
- Water
- Measurement Activity Record Sheet (1 per student)

Procedure:

1. Show video *The Great Pacific Garbage Patch* to introduce the environmental problem of plastics in the ocean (http://www.youtube.com/watch?v=y5y1W5xduiE). *You can use the free video conversion tool (www.keepvid.com) to download this video to your computer (480p mp4 format) for future use. The keepvid conversion also strips off the commercials from YouTube videos.*

2. Tell students that in this project they will begin to think about how the design of a water bottle can affect the plastic that ends up as trash.

3. Hand out a Measurement Activity Record Sheet to each student.

4. Next, students will experience comparing liquid measure to the volume that they calculate using formulas. Using a 1 Liter plastic water bottle, ask the students to measure 250mL of water into the bottle. After adding the water, students are to measure the diameter ($d = 2r$) of the bottle and the height of the *water column* and record this information on their Measurement Activity Data Sheet.

5. Provide students with the formula for the volume of a cylinder, $V = \pi r^2 h$. Ask groups to use the diameter, d, and the height of the water column, h, that they just measured from the bottle that was filled to 250 ml to calculate the volume of the cylinder. (Due to the irregular shape of the bottom of the bottles, or other irregularities of the bottle, our calculations use an "equivalent" cylinder for easy calculation using the formulas given in Figure 3. To approximately account for complex shape of the bottom of the bottle, subtract 0.5 cm from the height measurement to the top of the water before calculating S and V.)

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6. Ask students to calculate the surface area, S, of the bottle filled to 250mL using the formula, $S = 2\pi r^2 + 2\pi rh$ using the same d and h as they used to calculate the volume of the cylinder.

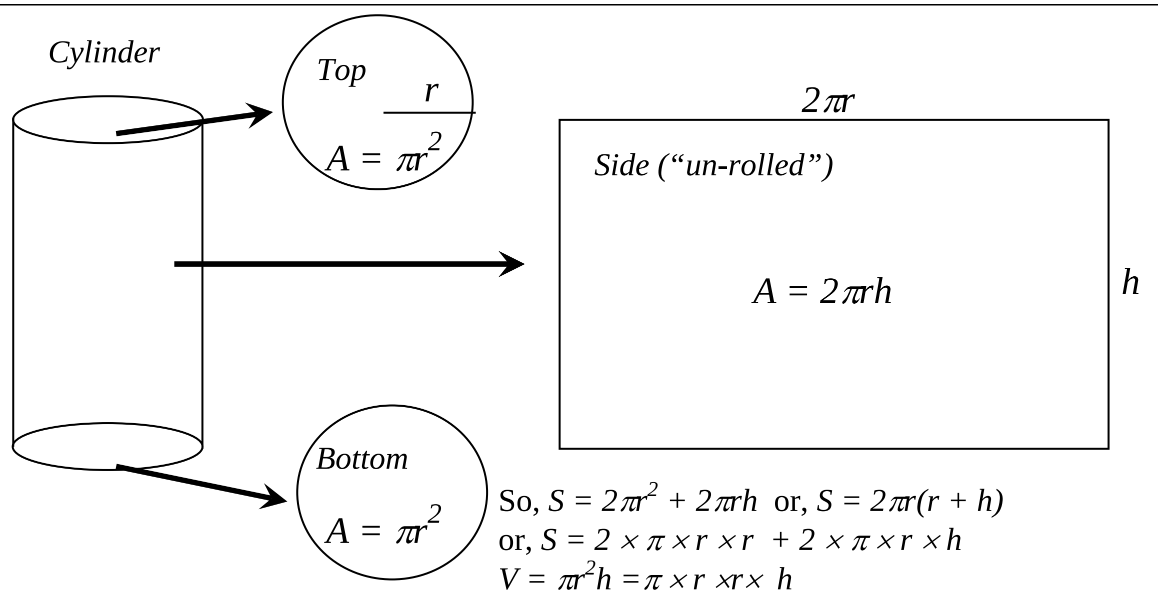
7. Now, ask students to calculate the surface area (S), volume (V) and S/V ratio for this sample of water and record this information on the Measurement Activity Data Sheet.
(Note: Students may use a calculator to find the decimal equivalence of this ratio.)

8. Repeat steps 4-7 using 500, 750 and 1000 ml as the measured volumes of water. Record information on the Measurement Activity Record Sheet.

9. Discuss with the students:
   a. How does the Amount of Water Added compare with the Volume?
   b. How does the S/V ratio vary with volume for this one bottle?

<u>Additional Notes for Teacher for Lesson 1:</u>

- Using the following diagram, students should learn to break a complex structure into known parts and then sum the surface areas of each part to calculate the total surface area.



So, $S = 2\pi r^2 + 2\pi rh$ or, $S = 2\pi r(r + h)$
or, $S = 2 \times \pi \times r \times r + 2 \times \pi \times r \times h$
$V = \pi r^2 h = \pi \times r \times r \times h$

r = radius, h = height, A = area, S = surface area

Figure 2. Calculating the surface area of a cylinder: 1) Calculate the surface area of the top of the cylinder; 2) Calculate the surface area of the bottom of the cylinder; 3) Calculate the surface area of the sides by unrolling the side into a rectangle; 4) Sum the surface area from steps1-3 to calculate the overall surface area of the cylinder.

- To approximately account for complex shape of the bottom of the bottle, subtract 0.5 cm from the height measurement to the top of the water before calculating S and V.

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- The three-dimensional concept can be further explored by using "nets" of the complex solids to make 3-D models: http://www.senteacher.org/wk/3dshape.php and http://www.korthalsaltes.com/index.html.

  Examples of nets for a right, closed cylinder and a rectangular prism are shown in Figure 3. Scaled versions of these nets are available at http://web.ics.purdue.edu/~braile/new/NetCylinder.docx and http://web.ics.purdue.edu/~braile/new/NetRectangularPrism.docx. If the nets are printed on 11 x 17 inch paper, the net will form 500 ml volume, 3-D shapes. The cylinder has a height that is twice the diameter. The rectangular prism has a square cross section ($l = w$) and the height is twice the width. After making the 3-D models, the students can measure the dimensions of the paper models and calculate the volume and the surface area of the shapes. These shapes are very similar to shapes used for plastic bottles.

- The students can also use paper to design or sketch two dimensional representations of three dimensional objects (examples in Figure 3 and online - http://www.korthalsaltes.com/pdf/rectangular_prism.pdf).

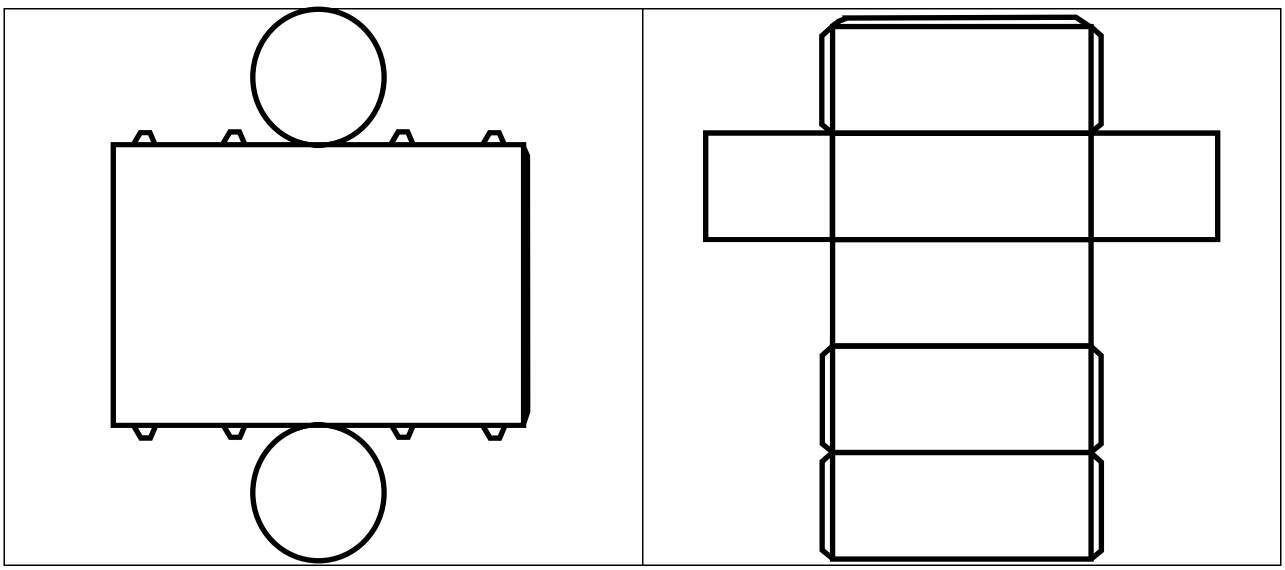

Figure 3. Left – Right, closed cylinder net. Right – Rectangular prism net. With full-size nets, cut out, fold, and use glue or tape on tabs to complete the 3-D shape.

Additional shapes (Figure 4) can be also be explored (with formulas for calculating volume and surface area).

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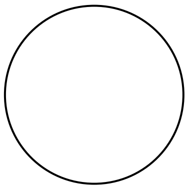
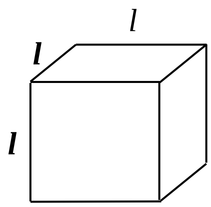
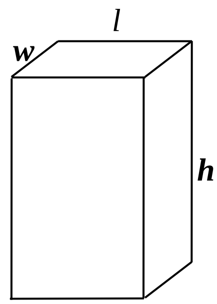
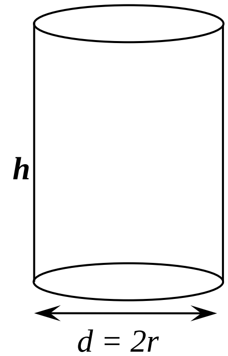
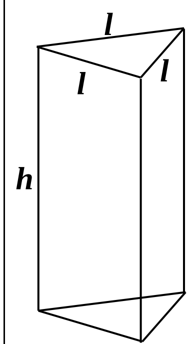
Sphere	Cube	Rectangular Prism	Cylinder	Triangular Prism*
				
$S = 4\pi r^2$ $= 4 \times \pi \times r \times r$ $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times r \times r \times r$	$S = 6l^2$ $= 6 \times l \times l$ $V = l^3$ $= l \times l \times l$	$S = 2lw + 2hw + 2lh$ $= 2 \times l \times w + 2 \times h \times w + 2 \times l \times h$ $V = lwh$ $= l \times w \times h$	$S = 2\pi r^2 + 2\pi rh$ $S = 2 \times \pi \times r \times r + 2 \times \pi \times r \times h$ $V = \pi r^2 h$ $V = \pi \times r \times r \times h$	$S = 0.866l^2 + 3lh$ $= 0.866 \times l \times l + 3 \times l \times h$ $V = 0.433\, l^2 h$ $= 0.433 \times l \times l \times h$

Figure 4. Surface Area and Volume calculations ofthree-dimensional shapes; d =diameter, r = radius, π = 3.1416, S = surface area, V = volume, l = length, w = width, h = height.
* Top and bottom are equilateral triangles.

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Lesson Plan #2
Guiding Question – Exploring Measurements of Cylindrical-type Water Bottles

Time: one 50 minute class session

Objectives: Students will consider the shapes of plastic water bottles currently used and predict which bottle would be the "best" bottle. Students will then perform mathematical calculations to determine which bottle uses the least amount of plastic (surface area of "equivalent" cylinder) per volume.

Materials:

A variety of cylindrical-type plastic water bottles (of varying shapes and volumes)
Plastic Used in Cylindrical-type Water Bottles Data Sheet (1 per student)
Calculators
Rulers (cm scale)

Procedure:

1. Show students various styles of water bottles that are available for purchase. These bottles should be of varying volumes (see Figure 5).

2. Ask students to predict which would be the "best" bottle in that it would use the least amount of plastic per volume.

3. Remind students of Lesson 1 and review the *S/V* ratio.

4. Hand out "Plastic Used in Water Bottles Data Sheet" to each student.

5. Ask each group to select one bottle that they think would have a small *S/V* ratio.

6. Students are to fill out the "Plastic Used in Cylindrical-type Water Bottles Data Sheet" for their bottle, using the given volume (from the label) and measuring the height to where water would be if it were full.

7. Next, students are to select a different bottle and repeat the same procedure for a designated time (perhaps 20 - 30 minutes). Groups are to select several bottles and perform the calculations.

8. When time is up, lead a discussion about which bottle uses the least amount of plastic and which bottle would be "best."

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Figure 5. Various Plastic Bottle shapes and Sizes.

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Lesson Plan #3
Guiding Question - Optimize the Shape and Size of the Plastic Bottle

Time: one 50 minute class session

Objective: Students will consider plastic water bottles that are made of shapes other than cylinders. They will calculate surface area and volume of these shapes and determine the ***S/V*** ratio in order to determine which shape is optimal in a water bottle design.

Materials:
- Calculator
- Optimization Data Sheet (two-sided worksheet)

Procedure:

1. Provide each student with an Optimization Data Sheet and a calculator. To complete this sheet, students must using the equations provided in Figure 4 to determine the surface area and volume for the sphere, cube, rectangular prism, and cylinder using $r = 1$ cm, $h = 1$ cm and $l = 1$cm.

2. Once groups have completed the calculations, lead class in discussion of the following questions.
   a. Which shape provides the best surface area to volume ratio? *(sphere)*
   b. What are the issues about using this shape? *(holding, setting down, drinkability)*
   c. Is another shape more practical? *(cylinder, rectangular prism)*

3. Next, using the cylinder shape and a set volume (500 ml), ask students to determine the optimal radius and height of the cylinder. (The "shape" of the cylinder – whether it is "short and fat" or "tall and skinny" – is sometimes referred to as the *aspect ratio* and is equivalent to the r/h value that we will use to compare shapes.) To do this, instruct students to first calculate the height of each cylinder, using the given radius the list of radius values to use is given below). Once they have determined the height, they can use that value, along with 3.14 for π, to determine the surface area of the cylinder. The equations/calculations are:

   $V = \pi r^2 h$ so, for a given volume (V = 500ml), and radius (r, trial value),
   $h = V/\pi r^2$ (calculate the value of h)

   then, $S = 2\pi r^2 h + 2\pi rh$ (calculate the value of S)

   so we can find an r and h that result in the smallest surface area (S).

   To do this, the student should calculate h and S for r = 1.5, 2.5, 3.5, 4.5, 5.5, 6.5 centimeters. Then calculate r/h and S for each of the six values of r and plot a graph of r/h versus S.

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4. Ask students to plot the data on the graph template below. They are to use the x-axis for the *r/h* and the y-axis for the surface area.

   Note to teacher: The result will be the red dots on the graph (see resources below) and the students will be able to connect the dots with a smooth line and determine that the optimum value of r/h (minimizes the surface area) is about 0.5 (this is actually the ratio of r/h that minimizes S for right, closed cylinders of any volume).

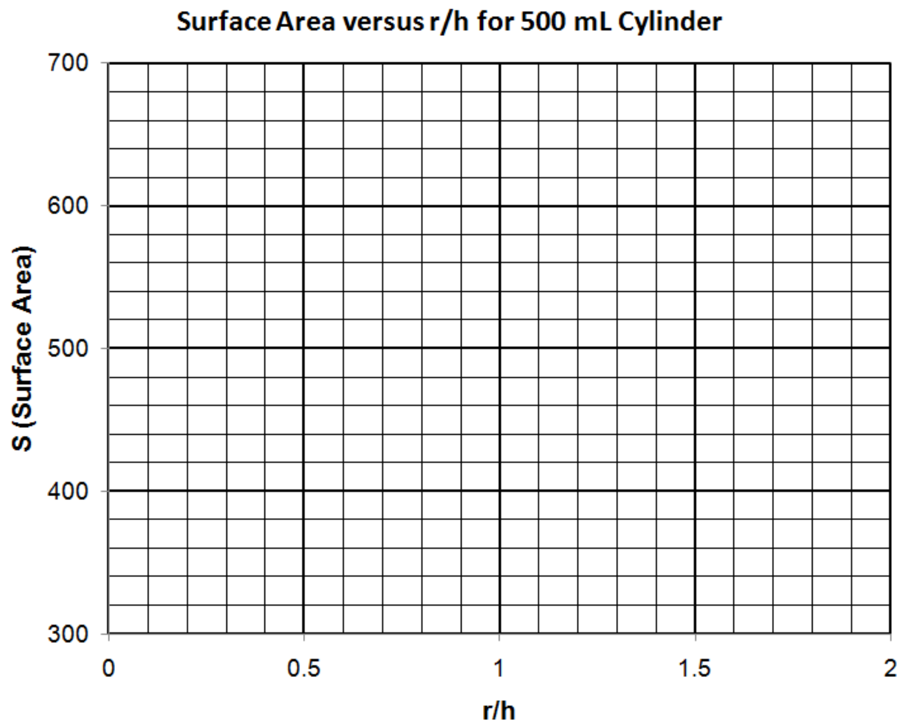

Figure 6. Template for graphing r/h versus S to find the optimum shape (aspect ratio) of a cylinder shaped bottle that has a volume of 500 ml.

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Lesson Plan #4
Design Challenge—Build a Better Bottle; What's the Best Shape?

Time: one 50 minute class session

Objective: Students will design a more efficient water bottle.

Materials:

Calculator
Design Challenge Sheet "Build a Better Bottle—What's the Best Shape?" (1 per group)
White paper (for the design, sketches and calculations)
Ruler

Procedures / Steps:

1. Provide a Design Challenge Sheet to each group.
2. Give students time to design their "better" water bottle.

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Build a Better Bottle – What's the Best Shape?
Student Resource

Image obtained from: http://veghunter.wordpress.com/2010/09/16/stainless-steel-water-bottles-are-they-better-than-plastic-and-glass/

Design and Sketch a Water Bottle; Calculate the *S/V*

Design Challenge: *"Can you design a more efficient water bottle?"*

A large number of plastic water bottles are used each year. However, these plastic bottles impact the environment – ending up in our oceans and taking a very long time to biodegrade in landfills. Several companies are taking steps to reduce their use of plastic by making the caps smaller, reducing the thickness of the plastic so less is used, and using plastic that is more biodegradable. Your class is being asked to contribute to these efforts by identifying a shape that maximizes the amount of liquid that can be stored but uses the least amount of plastic. Your team will need to sketch the water bottle and prove the efficiency of your design, as well as how it meets the other criteria below.

The water bottle should:

- Hold 500 ml of liquid
- Use the least amount (approximately) of plastic
- Able to be held in your hand
- Be designed so that a person is able to drink from it
- Stand upright/set on a table or desk
- Be able to be packaged/shipped/stored easily

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Lesson Plan #5
Design Challenge – *What feedback do you get on your design?*

Time: one 50 minute class session

Objective: Students will examine the bottle designs of their group as well as their classmates to determine if they have built a bottle that uses less plastic.

Procedures / Steps:

1. Allow time for students to look at the water bottle designs of all groups.

2. Lead class in discussion about the designs using the following questions:
   a. What are the conclusions of our analysis of shapes and plastic bottles?

   b. How much saving can be achieved by optimizing the shape? (one can calculate percent savings of one design versus another from surface area, for equivalent volumes)

   c. What are limitations of our analysis? (equivalent shape approximations, neglecting cap area, strength of materials or thickness of plastic, could use other materials, could use a more complicated shape such as a sphere with a flat area on the bottom so that it would stand up on a flat surface)

   d. What are other considerations in determining the "best" shape for a plastic bottle? (efficiency of manufacturing and shipping, marketability – how attractive it is to consumers, will container fit in cup holders in vehicles?)

   e. What other possible solutions are there to the problem of plastic bottles?

   f. Is the bottle design practical for a person using it?

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Assessment

The following are possible sources of **formative** and **summative assessment** (From the Evaluation level of Bloom's Taxonomy):

- Which plastic bottle design do you think is the best? Why do you think so?
- Judge which is the best solution to the problem of using less plastic in the design of water bottles to eliminate as much plastic waste as possible. Why do you think so?
- (From the Inference Level of Bloom's Taxonomy)
- Predict what would happen if you bottle was manufactured. How would it be shipped? How would it be displayed in stores?
- (From the Analysis Level of Bloom's Taxonomy)
- What is the relationship between the volume and surface area of a plastic water bottle?

Formative Assessments

- Teacher circulates the room and informally evaluates the work of each group, guiding them if needed in their calculations and graphing.Other (individual and/or group) – Create a short pre and posttest that highlights key science vocabulary terms; Present a new situation or new problem on the same theme.

Lesson Extensions and Resources

Activity Extensions: A possible extension is to investigate ocean currents and the collection of trash in the Pacific or Atlantic Oceans.

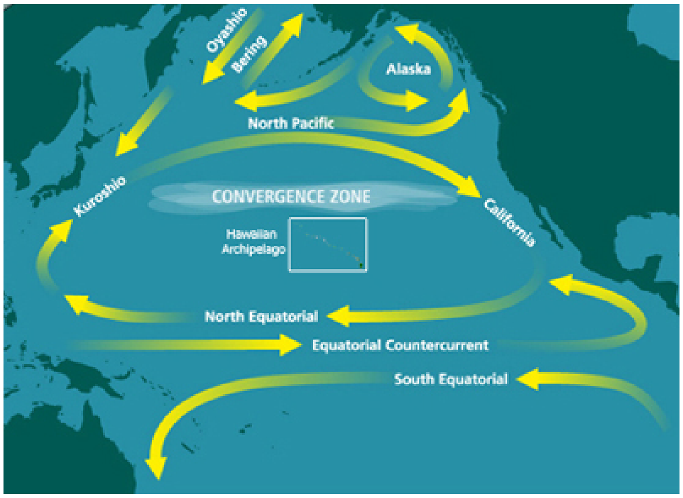

Figure 7. North Pacific Subtropical Convergence Zone.jpg, source Wikipedia.

Web Resources:

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Videos

1) The garbage patch:
    a) Shorter video (~2:45 min): http://www.youtube.com/watch?v=tnUjTHB1lvM
    b) Another video (~3:00 min): http://www.youtube.com/watch?v=y5y1W5xduiE
    c) Each good for lesson introduction
2) The great pacific garbage patch
    a) (Part1 – Longer video ~9:30 min): http://www.youtube.com/watch?v=ISaGrlpK2zE
    b) Discusses the impact of plastic on wildlife and the environment. Contains videos of animals either killed or maimed by plastic. Shows pre-production plastic pellets used to make plastic products that never get into the hands of consumers. (Very small aquatic wildlife could easily mistake for food).
3) Captain Charles Moore: The great pacific trash island: http://www.youtube.com/watch?NR=1&v=en4XzfR0FE8&feature=endscreen
    a) TED Talk: content similar to the first entry with a lot more statistics
4) The trash vortex
    a) Source – Greenpeace International: http://www.greenpeace.org/international/en/campaigns/oceans/pollution/trash-vortex/

Articles

5) The Pacific garbage patch explained
    a) Source - How stuff works: http://science.howstuffworks.com/environmental/conservation/issues/great-pacific-garbage-patch-explained.htm
6) The biggest dump in the world
    a) Source – The telegraph: http://www.telegraph.co.uk/science/7450769/The-Biggest-Dump-in-the-World.html
7) Demystifying the "Great Pacific Garbage Patch" or "Trash Vortex"
    a) http://axisoflogic.com/artman/publish/Article_55957.shtml

Other Resources:

http://www.doe.in.gov/sites/default/files/assessment/math-referencesheet.pdf - Reference for notation for volume and surface area formulas.

Mini-lesson #1 Volume Equal to 500mL.

Math Standards:

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CCSS 6.G.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

For this lesson students have the opportunity to design and construct a model of a plastic bottle that contains a volume of 500mL. Two possible shapes that can be used in the design are the rectangular prism ($V = l \times w \times h$) and the cylinder ($V = \pi \times r \times r \times h$).

Remind students that the volume (V) of a rectangular prism is found using the formula, $V = l \times w \times h$. Show students how to substitute 500 for the**V** in the equation. Once the volume is 500, students can see that the product of ***l***, ***w***, and ***h*** must equal 500. At this point, students can use calculators to investigate three measurements that multiply together to equal 500. These results can be used to determine the practicality of a water bottle with those dimensions. Students can then adjust their three choices to fit their criteria of what characteristics their water bottle should possess.

Next, remind students of the formula for the volume of a cylinder, $V = \pi \times r \times r \times h$. This time, the use of the formula is interesting. Students have to find three numbers (two of which are the same) that multiply with ***pi*** (3.1416) to equal 500. Students follow the same procedure as above to consider the characteristics important to the design of their water bottle.

Mini-lesson #2 Measuring the diameter of a sphere or cylinder.

Here are three techniques for measuring the diameter of a sphere or cylinder. This is a good challenge question for students.

Figure 8. Measuring the diameter of a cylinder (in this case a plastic water bottle): Left photo – place the cylinder flat on a table; place two books (rectangles) upright on the table and move them to touch opposite sides of the cylinder; measure the distance between the edges of the books with a ruler. Center photo – wrap a string around the cylinder to measure the circumference; divide the circumference by ***pi*** *(3.1416) to get the diameter. Use a caliper (such as the homemade one in the photo) to measure the width of the cylinder, then use a ruler to determine the distance between the two points on the caliper.*

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Figure 9. Other types and sizes of containers. Left – Juice box. Center – Milk carton. Right – Large (volume = 3.78 liters) plastic water bottle; would this bottle be more efficient (smaller ***S/V****) than smaller volume bottles?; the dimensions of an equivalent rectangular prism for this bottle are* ***l*** *=* ***w*** *= 13.2 cm,* ***h*** *= 21.6 cm.*

Figure 10. Various Plastic Bottle shapes and Sizes.

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Table 1. Surface area and volume of "equivalent cylinders" for selected plastic bottles.

Bottle (in photo)	**Size (ml)***	**r (cm, measured)**	**h (cm, measured)**	**S (cm^2)**	**V (cm^3)**	**S/V (1/cm)**
L	250	3.3	7.4	221.9	253.2	0.876
J	250	3.1	8.3	222.0	250.6	0.886
I	300	3.0	10.6	256.4	299.7	0.855
G	350	3.4	9.6	277.8	348.6	0.797
H	500	3.3	14.7	373.4	502.9	0.742
E	500	3.0	17.7	390.2	500.5	0.780
C	1000	4.2	18.0	585.8	997.5	0.587
B	1000	3.7	23.2	625.4	997.8	0.627
A	2000	5.5	21.0	915.8	1995.7	0.459
Bottle (in photo)	**Size (ml)***	**a (cm, measured)**	**h (cm, measured)**	**S (cm^2)**	**V (cm^3)**	**S/V (1/cm)**
K	500	6.0	13.9	405.6	500.6	0.811
D	1000	7.4	18.3	651.2	1002.1	0.650

* Size = bottle size (volume of liquid) on label, ml = milliliters, r = radius (for closed right cylinder shape), cm = centimeters, h = height, a = length of side (for rectangular prism shape), S = surface area of cylinder or rectangular prism, V = volume, S/V = surface area to volume ratio for approximately equivalent cylinder or rectangular prism shape.

Table 2. Calculations for optimization problem for graph shown in Figure 6.

r (cm)	**h (cm)**	**r/h**	**S (square cm)**
1.5	70.736	0.02121	680.8
2.5	25.465	0.09818	439.27
3.5	12.992	0.26939	362.68
4.5	7.8595	0.57256	349.46
5.5	5.2613	1.0454	371.88
6.5	3.767	1.7255	419.31

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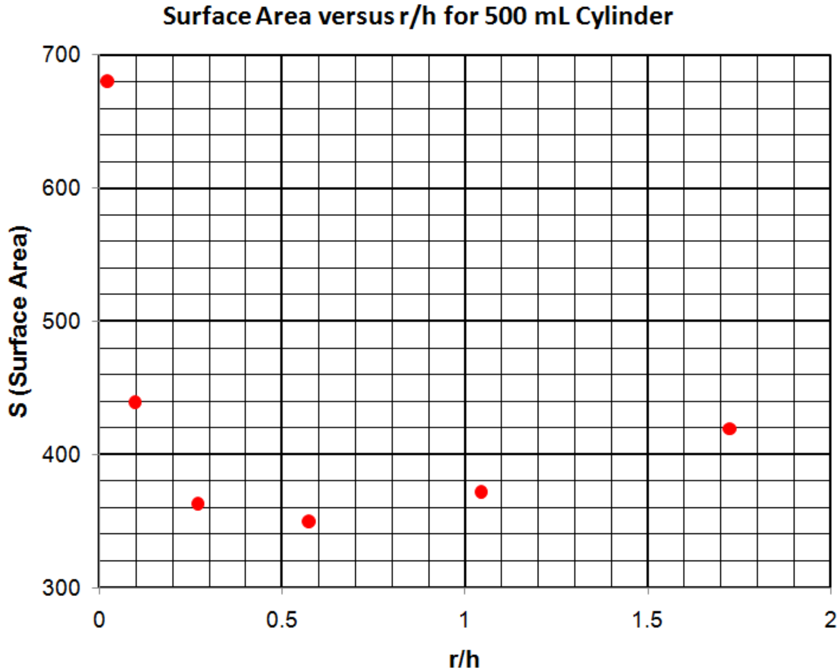
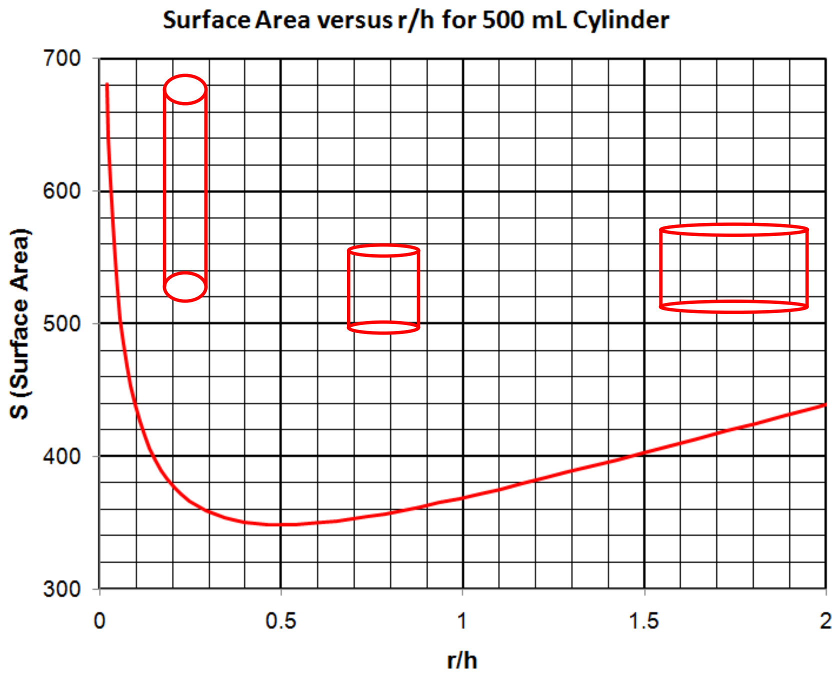
Graphing results:





Smooth line drawn through calculated points shows minimum (optimum or best r/h to use the least amount of plastic for a 500 ml cylinder). Cylinder diagrams illustrate regions of the curve where the cylinder is "tall and skinny" and where it is "short and fat."

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Student work pages:

Measurement Activity Data Sheet

Name___________________________________

Amount of Water Added to Bottle	**Radius of the Bottle**	**Height of the Water Column**	$V = \pi \times r \times r \times h$ **(Use 3.14 for π)**	$S = 2 \times \pi \times r \times r + 2 \times \pi \times r \times h$	S/V
250 ml					
500 ml					
750 ml					
1000 ml					

1. How does the Amount of Water Added compare with the Volume?

2. How does the S/V ratio vary (change) with volume for this one bottle?

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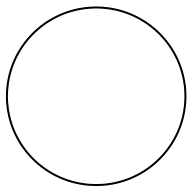
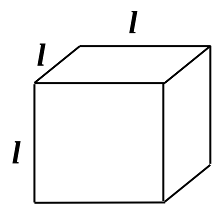
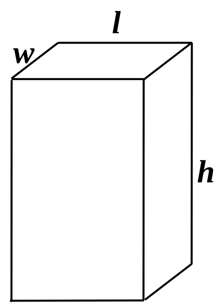
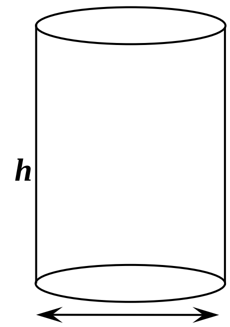
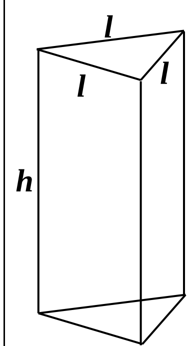
Optimization Data Sheet

Name________________________________

Determine the surface area and volume for the sphere, cube, rectangle, and cylinder using ***r* = *1*** cm, ***h* = *1*** cm and ***l* = *1*** cm.

Sphere	**Cube**	**Rectangular Prism**	**Cylinder**	**Triangular Prism***
			 $d = 2r$	
$S = 4\pi r^2$ $= 4 \times \pi \times r \times r$ $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times r \times r \times r$	$S = 6l^2$ $= 6 \times l \times l$ $V = l^3$ $= l \times l \times l$	$S = 2lw + 2hw + 2lh$ $= 2 \times l \times w + 2 \times h \times w + 2 \times l \times h$ $V = lwh$ $= l \times w \times h$	$S = 2\pi r^2 + 2\pi rh$ $S = 2 \times \pi \times r \times r + 2 \times \pi \times r \times h$ $V = \pi r^2 h$ $V = \pi \times r \times r \times h$	$S = 0.866l^2 + 3lh$ $= 0.866 \times l \times l + 3 \times l \times h$ $V = 0.433\, l^2 h$ $= 0.433 \times l \times l \times h$

** Top and bottom are equilateral triangles.*

	Sphere	**Cube**	**Rectangular Prism**	**Cylinder**
S				
V				
S/V				

1. Which shape provides the best surface area to volume ratio (uses the least amount of plastic)?

2. What are the issues about using this shape for a water bottle?

3. Which shape is more practical to use for the design of a water bottle?

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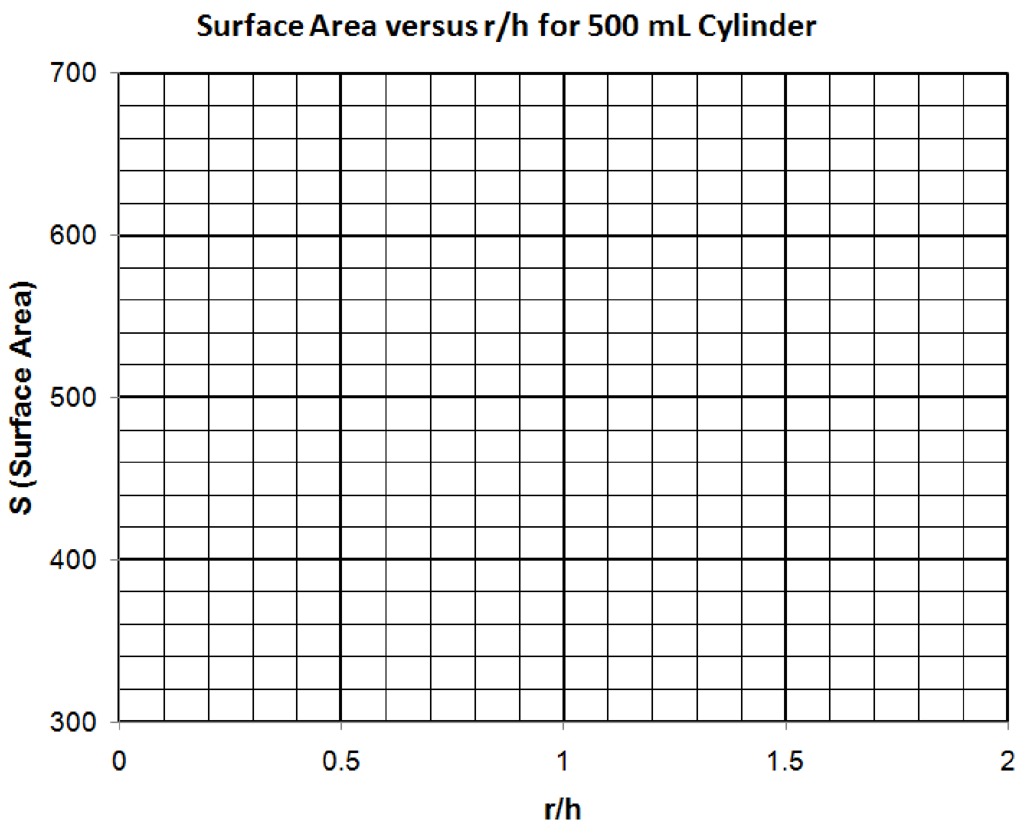
Optimization Data Sheet (continued)

Name________________________________

Use the five radii (***r***) given to determine the height of a cylinder (with ***V*** = 500 ml) with each of the radii. Then, write the ratio ***r/h***. Finally, using 3.14 for π, determine the surface area, ***S***, of each cylinder. You may use a calculator. Then, plot the points (***r/h*** versus ***S***) on the graph.

radius (*r*, cm)	**height (*h* in cm)** $h = V/\pi r^2$	***r/h*** **(no units)**	**surface area (*S*, in square cm)** $S = 2\pi r^2 h + 2\pi rh$
r **= 1.5 cm**			
r = 2.5 cm			
r = 4.5 cm			
r =5.5cm			
r =6.5cm			



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Plastic Used in Cylindrical-type Water Bottles Data Sheet

Name________________________________

Capacity of Water Bottle (in mL)	Radius of the Bottle	Height	$V = \pi \times r \times r \times h$ (Use 3.14 for π)	$S = 2 \times \pi \times r \times r + 2 \times \pi \times r \times h$	S/V

1. Which bottle do you think is the "best"? Why?

2. Which bottle uses the least amount of plastic?

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